数理解析研究所講究録 1222 巻 2001 年 46-49

CPN Languages and Codes

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Let $D=(P,X,\delta,\mu_0)$ be a Petri net with a initial marking μ_0 where P is the set of places, X is the set of transitions, δ is the transition function and $\mu_0\in N_+^P$ is a positive marking, i.e. $\pi_p(\mu_0)>0$ for any $p\in P$. Notice that $\pi_p(\mu_0)$ is meant the number of tokens at p of the marking μ_0. A language C is called a *CPN language* over X generated by D and denoted by $C=\mathcal{L}(D)$ if $C=\{u\in X^+|\exists p\in P,\pi_p(\delta(\mu_0,u))=0, \forall q\in P,\pi_q(\delta(\mu_0,u))\geq 0$, and $\forall q'\in P,\pi_{q'}(\delta(\mu_0,u'))>0$ for $u'\in P_r(u)\setminus\{u\}$ where $P_r(u)$ is the set of all prefixes of $u\}$. Then it is obvious that $C=\mathcal{L}(D)$ is a prefix code over X. If C is a maximal prefix code over X, then C is called an *mCPN langauge* over X.

Theorem 1 *Let $A,B\subseteq X^+$ be finite maximal prefix codes over X. If AB is an $mCPN$ language over X, then A,B are full uniform codes over X.*

Remark 1 In the above theorem, the condition for A and B to be finite is necessary. For instance, let $X=\{a,b\}$ and let $A=B=b^*a$. Then $AB=b^*ab^*a$ is an $mCPN$ language over X but neither A nor B is a full uniform code over X.

Now we consider some constructions of $mCPN$ languages.

Definition 1 Let $A,B\subseteq X^+$. Then by $A\oplus B$ we denote the language $(\cup_{b\in X}\{(P_r(A)\setminus A)\diamond Bb^{-1}\}b)\cup(\cup_{a\in X}\{(P_r(B)\setminus B)\diamond Aa^{-1}\}a)$ where $\diamond$ is meant the shuffle operation.

Proposition 1 *Let $X=Y\cup Z$ where $Y,Z\neq\emptyset, Y\cap Z=\emptyset$. If $A\subseteq Y^+$ is an $mCPN$ language over Y and $B\subseteq Z^+$ is an $mCPN$ language over Z, then $A\oplus B$ is an $mCPN$ language over X.*

Example 1 Let $X = \{a,b\}$. Consider $A = \{a\}$ and $B = \{bb\}$. Then both A and B are $mCPN$ languages over $\{a\}$ and $\{b\}$, respectively. Hence $A \oplus B = \{a, ba, bb\}$ is an $mCPN$ language over X.

Proposition 2 *Let $A, B \subseteq X^+$ be finite $mCPN$ languages over X. Then $A \oplus B$ is an $mCPN$ language over X if and only if $A = B = X$.*

Remark 2 For the class of infinite $mCPN$ languages over X, the situation is different. For instance, let $X = \{a,b\}$ and let $A = B = b^*a$. Then $A \oplus B = b^*a$ and A, B and $A \oplus B$ are $mCPN$ languages over X.

Proposition 3 *Let $A, B \subseteq X^+$ be $mCPN$ languages over X. Then there exist an alphabet Y, $D \subseteq Y^+$: an $mPCN$ language over Y and a homomorphism h of Y^* onto X^* such that $A \oplus B = h(D)$.*

Definition 2 Let $A \subseteq X^+$. By $m(A)$, we denote the language $\{v \in A | \forall u, v \in A, \forall x \in X^*, v = ux \Rightarrow x = 1\}$. Obviously, $m(A)$ is a prefix code over X. Let $A, B \subseteq X^+$. By $A \otimes B$, we denote the language $m(A \cup B)$.

Proposition 4 *Let A, B be $mCPN$ languages over X. Then, $A \otimes B$ is an $mCPN$ language over X.*

Example 2 It is obvious that a^*b and $(a \cup b)^3$ are $mCPN$ languages over $\{a,b\}$. Hence $a^*b \otimes (a \cup b)^3 = \{b, ab, aaa, aab\}$ is an $mCPN$ language over $\{a,b\}$.

Remark 3 Proposition 4 does not hold for the classe of CPN languages over X. The reason is the following: Suppose that $A \otimes B$ is a CPN language over X for any two CPN languages A and B over X. Then we can show that, for a given finite CPN language A over X, there exists a finite $mCPN$ language B over X such that $A \subseteq B$ as follows. Let $A \subseteq X^+$ be a finite CPN language over X which is not an $mCPN$ language. Let $n = max\{|u| | u \in A\}$. Consider X^n which is an $mCPN$ language over X. By assumption, $A \otimes X^n$ becomes a CPN language (in fact, an $mCPN$ language) over X. By the definition of the operation $\otimes$, it can be also proved that $A \subseteq A \otimes X^n$. Notice that there exists a finite CPN language A over X such that there exists no $mCPN$ language B over X with $A \subseteq B$. Hence, Proposition 4 does not hold for the class of all CPN languages over X.

Remark 4 The set of all $mCPN$ languages over X forms a semigroup under $\otimes$. Moreover, the operation $\otimes$ has the following properties:

$$(1)\ A \otimes B = B \otimes A,\ (2)\ A \otimes A = A,\ (3)\ A \otimes X = X.$$

Consequently, the set of all $mCPN$ languages over X forms a commutative band with zero under $\otimes$.

Definition 3 Let $A \subseteq X^+$ be a CPN language over X. By $r(A)$ we denote the value $min\{|P||D = (P, X, \delta, \mu_0), \mathcal{L}(D) = A\}$.

Remark 5 Let $A \subseteq X^+$ be a finite CPN language over X. Then $r(A) \leq |A|$. Moreover, let $A, B \subseteq X^+$ be $mCPN$ languages over X. Then $r(A \otimes B) \leq r(A) + r(B)$. In the above, if $|A|, |B|$ are finite, then $|A \otimes B| \leq max(|A|, |B|)$.

We define three language classes as follows: $\mathcal{L}_{CPN} = \{A \subseteq X^+ | A$ is a CPN language over $X\}$, $\mathcal{L}_{mCPN} = \{A \subseteq X^+ | A$ is an $mCPN$ language over $X\}$, $\mathcal{L}_{NmCPN} = \{A \subseteq X^+ | A$: an $mCPN$ language over X, $\exists D = (P, X, \delta, \mu_0), \forall p \in P, \forall a \in X, \#(p \to a) \leq 1, \mathcal{L}(D) = A\}$. Then it is obvious that we have the following inclusion relations: $\mathcal{L}_{NmCPN} \subseteq \mathcal{L}_{mCPN} \subseteq \mathcal{L}_{CPN}$. It is also obvious that $\mathcal{L}_{mCPN} \neq \mathcal{L}_{CPN}$.

Problem 1 $\mathcal{L}_{mCPN} \neq \mathcal{L}_{NmCPN}$?

Proposition 5 *Let $A \in \mathcal{L}_{CPN}$ and let $r(A) = k$. Then there exist $A_1, A_2, \ldots,$ $A_k \in \mathcal{L}_{CPN}$ such that $r(A_i) = 1, i = 1, 2, \ldots, k$ and $A = A_1 \otimes A_2 \otimes \ldots \otimes A_k$. Moreover, in the above, if $A \in \mathcal{L}_{NmCPN}$, then $A_1, A_2, \ldots, A_k$ are in $\mathcal{L}_{NmCPN}$ and context-free.*

For $mCPN$ languages with rank 1, we have the following:

Proposition 6 *Let $A \subseteq X^+$ be a finite $mCPN$ language with $r(A) = 1$ over X. Then A is a full uniform code over X.*

Proposition 7 *Let $A \subseteq X^+$ be an $mCPN$ language with $r(A) = 1$ over X and let k be a positive integer. Then A^k is an $mCPN$ language with $r(A^k) = 1$ over X.*

Proposition 8 *Let $A \in \mathcal{L}_{NmCPN}$ and let $r(A) = k$. Then there exist $A_1, A_2, \ldots, A_k \in \mathcal{L}_{NmCPN}$ such that $r(A_i) = 1, i = 1, 2, \ldots, k$ and $A =$*

$A_1 \otimes A_2 \otimes \ldots \otimes A_k$. *Let* $n_1, n_2, \ldots, n_k$ *be positive integers. Then* $A_1^{n_1} \otimes A_2^{n_2} \otimes \ldots, \otimes A_k^{n_k} \in \mathcal{L}_{NmCPN}$.

Finally, we can prove the following main theorem by two different ways, i.e. the first one is an indirect proof and the second one is a direct proof.

Theorem 2 *Let* $C \subseteq X^+$ *be a CPN language over* X. *Then* C *is a context-sensitive language over* X.